Atelier franco-japonais sur

Real and Complex Dynamics of Hénon's maps

Dates: From 9:00, March 27 to 17:00, March 31, 2023
Venue: Room 3–110, Department of Mathematics, Kyoto University

Program:

27 March (Monday)

9:00 – 9:45	Shigehiro Ushiki (Kyoto)
10:00 – 10:45	Shin Kiriki (Tokai)
11:00 – 11:45	Sébastien Biebler (Sorbonne)
13:30 – 16:30	discussion
16:30 – 17:15	Hiroki Takahasi (Keio)

28 March (Tuesday)

9:00 – 9:45	Raphaël Krikorian (École Polytechnique)
10:00 – 10:45	Mao Shinoda (Ochanomizu)
11:00 – 11:45	Takumi Yagi (Kyoto)
13:30 – 16:30	discussion
16:30 – 17:15	Ziyuan Zhang (CNRS – Paris 13)

29 March (Wednesday)

9:00 – 9:45	Reimi Irokawa (NTT)
10:00 – 10:45	Bernhard Reinke (CNRS – Sorbonne)
11:00 – 11:45	Thomas Richards (Warwick/Kyushu)
	free afternoon

30 March (Thursday)

9:00 – 9:45	Sylvain Crovisier (CNRS – Paris Saclay)
10:00 – 10:45	Yuki Hironaka (Kyushu)
11:00 – 11:45	Mathieu Helfter (CNRS – Sorbonne)
13:30 – 16:30	discussion
16:30 – 17:15	Zin Arai (Chubu)
18:00 –	banquet

31 March (Friday)

9:00 – 9:45	no talk
10:00 – 10:45	Hiroyuki Inou (Kyoto)
11:00 – 11:45	Johan Taflin (Bourgogne)
13:30 – 17:15	discussion

Titles and abstracts:

Shigehiro Ushiki (Kyoto)

Exotic rotation domains in complex Hénon dynamics.

Fatou component of complex dynamical system is called a rotation domain if the dynamics in the set is quasiperiodic. The closure of the orbit of almost any initial point is a circle or a torus. We say a rotation domain is exotic if the domain is not simply connected. In this talk, we explain how to observe such object numerically.

Shin Kiriki (Tokai)

$C^{1+\alpha}$-robust tangencies and pluripotent wandering domains for surface diffeomorphisms.

In this study we introduce the property of pluripotency. This means that arbitrarily small perturbations can stochastically realize arbitrary dynamics, including arbitrary recurrent dynamics or combinations thereof, arbitrary Dirac physical measures as well as historic behavior. In other words, it can be thought of as a stochastic version of universal dynamics. The pluripotent property has been suggested that this property also exists in 2-dimensional C^2 Newhouse domains [Colli–Vargas '01, Kiriki–Soma '17] and in the 3-dimensional C^1 domains [arXiv 2107.09844]. In this study we first prove the existence of a 2-dimensional $C^{1+\alpha}$ Newhouse domain O, which is folklore, and besides show the existence of a persistent class of O whose every element has a pluripotent wandering domain.

Sébastien Biebler (Sorbonne)

Typicality of the Newhouse phenomenon.

In the 70s, Newhouse discovered one of the most mysterious phenomenon in the field of dynamical systems: the existence of locally generic sets of dynamics displaying infinitely many sinks accumulating a Smale's horseshoe. Since then, many works studied to what extent the Newhouse phenomenon is typical. In this talk, I will precise what is meant by "typical" and I will discuss a joint work in progress with Pierre Berger. One of the main tools will be the notion of parablender.

Hiroki Takahasi (Keio)

On the LGYK conjecture on bifurcations of the real Henon family with small Jacobian.

Lai, Grebogi, Yorke, Kan [Nonlinearity, 1993] performed a crude numerical experiment on the dynamics near the bifurcation parameter of the real Henon family from Smale's horseshoe, and conjectured that "most" parameters from the bifurcation before the creation of the global chaotic attractor correspond to

a non-hyperbolic chaotic transient (in particular, no attractor). I will roughly explain how to prove a weaker version of this conjecture asserting that "most" parameters correspond to a chaotic transient. To my knowledge, the hyperbolicity or non-hyperbolicity of the transient remains essentially open.

Raphaël Krikorian (École polytechnique)

KAM theory of elliptic equilibria.

In this talk we shall review some basic results of KAM theory for complex symplectic mappings having an elliptic equilibrium (for example an elliptic fixed point). After the introduction of Birkhoff Normal Forms, I will state and give (a flavor of) the proof of a complex KAM theorem under some twist condition. Some applications of this KAM theorem will be given. I will then address the problem of lack of twist and discuss Herman's conjecture on the possibility of a KAM result without twist.

Mao Shinoda (Ochanomizu)

Ergodic optimization and rotation sets for symbolic dynamical systems

Ergodic optimization is the study of maximizing measures which are invariant measures with the maximum ergodic average for a given potential and has attracted recent research interest. Garibaldi and Lopes introduced rotation numbers as constraints in Ergodic optimization and prove fundamental properties in this context such as generic uniqueness. In this talk we consider generic zero entropy property in this constrained case, which is also an analogous problem with a unconstrained Ergodic optimization.

Takumi Yagi (Kyoto)

An approach to non-radial perturbations of semi-parabolic Hénon maps

We consider the Hénon maps given by $H(x, y) = (x^2+c+ay, ax), (c, a) \in \mathbb{C}^2$, where $|a|$ is sufficiently small. Hénon maps with semi-parabolic fixed points are not hyperbolic, difficult to deal with dynamically, and the Julia sets non-trivially vary on a neighborhood of a semi-parabolic parameter. On perturbations $\{H_t\}_{t\in[0,\epsilon]}$ of such Hénon maps, we see bifurcation phenomena where semi-parabolic fixed points bifurcate into fixed points and periodic points. We see that if the eigenvalues $\lambda_t, t \in [0, \epsilon]$ at bifurcated fixed points, satisfying that $\lambda_0^m = 1$ for some $m \in \mathbb{N}$ and $|\lambda_t| \neq 1$ for $t \neq 0$, have some nice condition, then the Julia sets vary continuously. Radu and Tanase showed if $\lambda_t = (1 \pm t)\lambda_0$, then the Julia sets vary continuously and $H_t, t \neq 0$ are hyperbolic. Let $\lambda_t/\lambda_0 = \exp(L_t + i\theta_t)$ and $\theta_t \to 0$ as $t \to 0$. We say that $\{H_t\}$ is a *radial perturbation* if $\theta_t = O(L_t)$. The results above hold if $\theta_t = O(L_t)$. In this talk, we introduce an approach to non-radial perturbations.

Zhiyuan Zhang (CNRS – Paris 13)

Newhouse phenomenon in the complex Hénon family.

In a work in progress with Avila and Lyubich, we show that there are maps in the complex Hénon family with a stable homoclinic tangency. This is due to a new mechanism on the stable intersections between two dynamical Cantor sets generated by two classes of conformal IFSs on the complex plane.

Reimi Irokawa (NTT)

Non-archimedean and hybrid dynamics of Hénon maps.

To study the meromorphic degeneration of dynamics, the theory of hybrid spaces, established by Favre, is known to be a strong tool. In this talk, we apply this theory to the dynamics of Hénon maps; for a family of Hénon maps $\{H_t\}_t$ parametrized by a unit punctured disk meromorphically degenerating at the origin, we show that as $t \to 0$, the family of the invariant measures $\{\mu_t\}$ "weakly converges" to the measure on the Berkovich affine plane which is naturally defined by the family $\{H_t\}_t$, in the sense of the theory of hybrid spaces.

Bernhard Reinke (CNRS – Sorbonne)

Emergence of random tree automorphisms

Emergence is way to describe the richness of possible statistical behaviors of orbits of a dynamical system. While it is known that high emergence is locally C^r generic for surface diffeomorphisms, it is less clear for "random" systems. We construct examples of conjugacy classes of rooted tree automorphisms such that the action of a random element of the conjugacy class on the ends of the tree has high emergence almost surely.

Thomas Richards (Warwick/Kyushu)

Monodromy and complex Hénon maps.

In 1991, Blanchard, Devaney, and Keen proved that loops in the shift locus of degree d polynomials induce all automorphisms of the one-sided shift on d symbols. Hubbard then conjectured that an analogous result holds for complex Hénon maps. His student, Chris Lipa, investigated this conjecture experimentally, proposing a relationship between loops around certain empirically observed components of the horseshoe locus and the combinatorics of the Mandelbrot set.

In this talk, we will discuss the conjectures and describe a new characterisation of the relevant parameter space components. We will also describe some experimental work aiming to understand the mechanism of monodromy in Hénon parameter space. Both of these utilise a new concept of critical point

for Hénon maps which mimics the behaviour we see in the one-dimensional monodromy problem.

Sylvain Crovisier (CNRS – Paris Saclay)

Existence of physical measures for smooth surface diffeomorphisms.

Marcelo Viana has conjectured that a smooth diffeomorphism admits a physical measure if the Lyapunov exponents of its orbits in a full volume set do not vanish. I will explain how a technique controlling the continuity of Lyapunov exponents allows to prove this conjecture in the case of smooth surface diffeomorphisms. This is a joint work with Jérôme Buzzi and Omri Sarig.

Yuki Hironaka (Kyushu)

Symbolic dynamics for Hénon maps near the boundary of the horseshoe locus.

In this talk, we consider the real Hénon maps $f(x, y) = (x^2 - a - by)$, where the parameter (a, b) is taken near the boundary of the horseshoe locus. We characterize their dynamics in terms of a symbolic dynamics. Concretely, we take a partition by two regions and consider the symbolic dynamics for this partition. Then, one can classify the behavior of Hénon maps according to the number of points which have a certain coding. Our result is an extension to any $b \neq 0$ of the previous one of Bedford and Smillie only for $b > 0$ close to zero. This is a joint work with Yutaka Ishii.

Mathieu Helfter (CNRS – Sorbonne)

Scales.

Scales propose a generalization of a part of dimension theory. Scales are finite bi-Lipschitz invariants that are defined on eventually infinite dimensional spaces. The comparisons between the different kind and growth of scales allow to describe the largeness of ergodic decompositions and functional spaces; or to study the behavior of the Wiener measure on small balls.

Zin Arai (Chubu)

On the disconnectedness of the Julia set for the Hénon map.

In this talk, we propose a rigorous algorithm for proving the disconnectedness of the Julia set for the complex Hénon map. In the case of the one-dimensional quadratic map, it is well known that the Julia set is connected if and only if the orbit of the critical point of the map is bounded. In our case of the Hénon map, since the map has no critical point, we need to keep track of a critical point of the Green function associated with the map. For this purpose, we develop a topological method that uses the plurisubharmonic nature of the Green function. This is a joint work with Yutaka Ishii.

Hiroyuki Inou (Kyoto)

A hole of the support of the bifurcation measure for the biquadratic family.

We numerically computed the bifurcation measure for the family of biquadratic polynomials. Through 4D rotation in VR visualization, we found a "hole" in the support. In this talk, we want to discuss how to check its existence by rigorous computation.

Johan Taflin (Bourgogne)

Blenders and the sparsity of postcritically finite endomorphisms.

Postcritically finite rational maps are of particular importance in one-variable complex dynamics. They are related to strong bifurcations phenomena and they form a Zariski dense set in the moduli space of degree d rational maps. In higher dimensions, we prove with Thomas Gauthier and Gabriel Vigny that postcritically finite maps are not Zariski dense in the moduli space of degree d endomorphisms of the projective space $\mathbb{P}^k$ as soon as d and k are larger or equal to 2. The proof is a combination of arguments coming from complex analysis, arithmetic geometry and smooth dynamics. An important step is to obtain an open set of maximal bifurcations using special hyperbolic sets called blenders.

Organizers:

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